Relational Approach to Wave-Particle Duality

Qianbing Zheng[†], Stephen Hughes[††], and Takayoshi Kobayashi

Department of Physics, University of Tokyo,

7-3-1 Hongo, Bunkyo-Ku, Tokyo 113, Japan

† *Department of Physics, Ohio State University,*

Columbus, Ohio 43210-1106 USA

†† *Department of Physics, Washington State University,*

Pullman, Washinton 99164-2814 USA

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Abstract

A relational approach to quantum optics is developed which regards the detection event as establishing a relationship between the quantized light field and the detector. The inherent ambiguity associated with applying Heisenberg's uncertainty principle is subsequently avoided.

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As is well known, Einstein's theory of relativity, which involves a profound analysis of time and space, introduced some radical changes, not only in our basic concepts, but also in our modes of physical reasoning. The essence of Einstein's theory was to adopt a relational approach to the notions of time and space, [1,2] which mathematically can be expressed through the Lorentz space-time transformations.

Although the mathematical structure of the Lorentz theory, which leaves the speed of light *in vacuo*, c, a universal constant, is equivalent to that of Einstein's, there is nevertheless a drastic difference in the way to conceive it. On the one hand, Lorentz began with retaining the customary concepts of absolute time and space of the older Newtonian mechanics, and by considering changes in the observing instruments. The invariant nature of c, as measured experimentally from the Michelson-Morley experiment, was successfully explained by the so-called 'Lorentz contraction', moving through the hypothetical *ether*. However, this theory led to the difficulty that the exact values of the 'true' distances and times, with respect to a detection scheme at rest in the ether, became somewhat ambiguous and unknowable. Einstein, on the other hand, by commencing with the observed facts, regarded time and space *a priori* as a certain class of 'coordinates' merely expressing *relationships* of an event to the measuring instruments. On the basis of a constant speed of light, both time and space become *relative* concepts, fundamentally dependent of the observer.

The developments of quantum formulation early this century has also led physicists to question the Newtonian concepts of physical objects, such as 'particle' and 'wave', which are basic ideas in all of classical physics. Subsequently, Heisenberg in his pioneering paper [3] developed a conceptual framework that in a way retained all the classical concepts, and plays a great role in the Copenhagen interpretation. This basic new step was to study the disturbance of observing instruments, and for this purpose, Heisenberg constructed the famous *gedanken* microscope experiment to measure very accurately the position of an electron. It was found that since the individual quanta of action must be taken into account in the measurement process, the irreducible disturbance rendered it impossible to assign *simultaneously* the precise values of position and momentum. Consequently, by considering the uncontrollable influence from the observation itself, the notion of particle into quantum mechanics was preserved, and the uncertainty principle was born.

In spite of its successes however, the Heisenberg theory has also brought about the problem, in a similar manner to the Lorentz theory, [1] that the *fundamental concepts*, e.g. the notion of particle in the interpretation, are in fact *completely ambiguous*. For it is deduced on the basis of the Heisenberg uncertainty principle that no means could ever give precisely a 'true' particle simultaneous values of position and momentum. This has been the object of severe criticisms from some other famous physicists, like Einstein, who has always believed that even in quantum theory there must exist precisely definable elements or dynamical variables to determine the actual behavior of each individual system. [4] In view of this fundamental ambiguity, it seems evident that a careful analysis of the notion of particle based on the actually measured facts is required, in parallel to Einstein's analysis of time.

In this letter we develop a relational approach to wave-particle duality which avoids the ambiguity associated with the Heisenberg theory. We emphasis, in parallel with Einstein's theory of special relativity, that for the proper analysis of quantum optics measurements with different frames of detection, one must consult a conceptual map of events which takes into account the perspective of the observer implicitly. The importance of events in quantum theory has been emphasized recently, [5,6] which for quantum optics can be described mathematically in terms of light detection. [7]

We begin by defining the *interaction* as establishing the presence of a physical object in which detection events serve as *relationships* between the object and the class of the measuring instrument. In other words, all our actual knowledge of a physical object is based on, at least in principle, the experimentally detected relationships between the object and a suitable detector.

In the quantum theory of radiation, the electric field operator in the Coulumb gauge may be written as the sum of positive and negative frequency parts

$$E(\mathbf{r},t) = E^{(+)}(\mathbf{r},t) + E^{(-)}(\mathbf{r},t), \tag{1}$$

where

$$E^{(-)}(\mathbf{r},t) = E^{(+)}(\mathbf{r},t)^{\dagger}. \tag{2}$$

One may expand $E^{(+)}(\mathbf{r},t)$ in terms of the normal modes as follows:

$$E^{(+)}(\mathbf{r},t) = i\sum_i [\frac{\hbar\omega_i}{2}]^{1/2} \hat{a}_i \varepsilon_i e^{i(\mathbf{k}_i\cdot\mathbf{r}-\omega_i t)}, \tag{3}$$

where ε_i are the unit vectors of polarization; this expansion has the same form as the classical expansion except that now the field amplitudes $\hat{a}_i$ are operators.

Glauber has studied the way in which light is detected, (7) and showed that, for an ideal photodetector situated at a point $\mathbf{r}$ in a radiation field, the probability of observing a photoionization event in this detector between time t and $t + \mathrm{d}t$ is proportional to $W_I(\mathbf{r},t)\mathrm{d}t$, where

$$W_I(\mathbf{r},t) =< \psi \mid E^{(-)}(\mathbf{r},t)\cdot E^{(+)}(\mathbf{r},t) \mid \psi >, \tag{4}$$

and $\mid \psi >$ specifies the state of the field. If one considers the one-dimensional propagation problem of one photon states: (8)

$$\mid \psi > = \sum_k c_k \hat{a}_k^{\dagger} \mid 0 >, \tag{5}$$

and

$$E^{(+)}(x,t) = \sqrt{\frac{\hbar c}{2}} \sum_k \sqrt{k}\hat{a}_k^{\dagger} e^{i(kx-\omega t)}. \tag{6}$$

Subsequently, the detection probability propagating along the x direction becomes:

$$W_I(x,t) = \frac{\hbar c}{2} \mid \sum_k \sqrt{k} c_k e^{i(kx-\omega t)} \mid^2 . \tag{7}$$

This probability of observing photoionization *in* detectors also reproduces the probabilistic wave of quantum phenomena. The Glauber detection theory differs from the Born probabilistic interpretation, (9) in that it expresses the meaning of physical law in terms of relationships, counting signals in the detection processes, without assuming the particle model of matter. These concepts quite naturally lead to a relational approach for the notion of physical object, and one can say that, in terms of actually measurable counting signals, the detection events follow laws of probability.

Here, we do not regard the above result as a deduction from the Heisenberg theory, but as a *basic hypothesis* which is well established experimentally. This needs little explanation, e.g. in terms of the disturbance of instruments, but is merely our starting point for further analysis; as in Einstein's theory of special relativity, we start from the *fact* that the speed of light is a constant.

We will consider the position measurement for an object, in order to see more clearly what this hypothesis implies with regard to the notions of *localizability* in physics, in a similar way as the discussion of simultaneity in Einstein's theory of special relativity. [(1)]

In Newtonian mechanics, one can of course mark the position for an object with the aid of a detector. The outcome of a detection in the system (between a detector and an object), or the occurrence of a detection event at a point in space indicates the position of the object. But as far as Newtonian mechanics is concerned, it is assumed that there is in essence only one position corresponding to an object. This implies that given any detection event at a position, as registered by an accurate detector, the other detection outcomes with the same procedures will be all co-located at the same point in space, as the first mentioned event, for the assemble of such position measurement. As a result, no detector that carries out the proper detection for the position measurement will ever find that any one of this set of events is different from each other at the location in space. If this is the case, then it makes sense to ascribe a definite position to the object, and to say that the object is localized at a point in space.

In quantum theory, however, the similar situation, for instance, the detection of light is described by the idea of one photon states, shown by Eq. (5). From a general property of Fourier transforms, the wave packet at a given time $E^{(+)}(x,0)$, with a spectrum width Δk, indicates that the detection of an event can no longer be localized to a specific point in space - which one assigns as a definite position to an object - but covers a range specified by Δx, where

$$\Delta k \ \Delta x \geq 1. \tag{8}$$

This is a major break with older ideas, because different detection in the assemble does not agree on what is the same position for an object. It must be emphasized, however, that whether localizability can be established is based only on an *indirect deduction*, the result of a statistical assembly, which expresses the deviation for the detection. Localizability is therefore no longer an *immediate fact* by which an object can be simplified as a point mass condensed at a spot in space in our everyday experience. For it is now seen to depend, to a large extent, on a purely conventional means of taking into account the deviation of the detected signals. This convention seems natural and inevitable to our common sense, but it leads to unambiguous results, an definite position for a physical object, only under conditions in which Newtonian mechanics is a good approximation. When the characteristic widths Δx and Δk can no longer be regarded as effectively infinite, then the experimental facts of physics make it clear that the results will depend on the characteristic widths for the problem in question.

It follows from the above discussion that localizability is not an *absolute* quality of objects, rather, its significance is dependent of characteristic widths of the discussed problem.

Consequently, although the mathematical structure of the above relational approach is equivalent to that of the Heisenberg theory which leads to the uncertainty principle, the underlying conceptual framework is vastly different. In the Heisenberg theory one deduces the uncertainty relation as a consequence of the disturbance of observing instruments as they are irreducibly participating in the observation, and subsequently, infer that a causal description is impossible for quantum theory, (however, Heisenberg's position seems incorrect, since the causal interpretation of quantum theory is, for example, successfully established by Bohm and his co-workers [4,11–13]); Δx is therefore interpreted as the uncertainty of position. On the contrary, by adopting a relational approach one begins with the experimentally wellconfirmed hypothesis of the probability of detection events, as *actually observed*. With this starting point, the above inequality implies that the concept of absolute position is no longer meaningful in quantum theory, where Δx specifies the deviation of detection. Indeed, once it is clear that the absolute position underlying localizability is not valid in quantum mechanics, it immediately follows that new concepts are needed to describe quantum processes, which contain the particle as a limiting case.

We have discussed that an outcome of detection (an event) specifies only a relationship between that object and a certain detection; however it is not suffice to consider only the result of an individual detection. The real significance of our detections arises from the fact that the properties of physical objects can be regularized and ordered in terms of *frames of detection*. For example, in a particle detection frame of light, one arranges a series of photodetectors in the propagation direction, by which one can define *invariant quantities* such as the velocity of the light signal propagation c (emission and absorption). This allows one not only to establish a 'trajectory' but also relate it to a portion of energy, E, and momentum, p, (a photon), transferred from a light field to a detector, to form a *particle picture* ($p = E/c$).

There also exists a wave frame of detection, where, for example, light is divided into two paths so as to interfere with each other. To measure and analyse such an effect, one also needs to place an array of detectors on the interfering plane, from which one can infer an additional set of quantities such as the frequency, wavelength, and also the phase velocity from the interference fringes; thus one constructs a *wave picture*. However, as far as Newtonian mechanics is concerned, such a wave frame of detection seems to be not necessary, and with the localizability discussed above, it makes sense to ascribe only the concept of particle to the cases investigated in the Newtonian domain.

Of course, all this experience depends on the condition that the de Broglie wavelength is so small that on the ordinary scale of distance and time, the wave modulation in the detection can be neglected; this is equivalent to assuming an infinitely small wavelength of matter. When a finite de Broglie wavelength is taken into account, new problems of 'wave-particle duality' do in fact arise, which ran through the famous Bohr-Einstein debate and is still a key issue in recent discussions. [14,15]

In terms of detection frames, the implications of the relational approach implies that there is infact no absolute significance to particle and wave pictures, but rather, their meaning is fundamentally dependent on how a frame of detection is constructed, i.e., on the observer. However, this concept of 'relativity', can only be expressed in precise quantitative form by Glauber's theory of light detection that logically unifies the two pictures of particle and wave.

From the relational viewpoint, physical phenomena in the quantum theory of light detection are described in terms of *fields* [Eq. (1)] and their *detection* [Eq. (4)], which are organized, ordered, and structured so as to correspond to the characteristics of radiation systems that are being studied. In the aforementioned theory, de Broglie's concepts are now manifested by $E(\mathbf{r},t)$, in terms of annihilation operator $\hat{a}_i$ (and creation operator $\hat{a}_i^\dagger$) as field amplitudes modulated by phase factors $e^{i(\mathbf{k}_i\cdot\mathbf{r}-\omega_i t)}$ (and the conjugated $e^{-i(\mathbf{k}_i\cdot\mathbf{r}-\omega_i t)}$). The key point that we wish to establish is that $E(\mathbf{r},t)$ contains information concerning the propagation properties of light in both the particle and wave frames of detection since on the one hand, the propagation characteristics of the operators $\hat{a}_i$ and $\hat{a}_i{}^\dagger$, which physically describe the absorption and emission of light, indicate a particle frame of detection where the light signal travels at the speed c. On the other hand, the phase factor $e^{i(\mathbf{k}_i\cdot\mathbf{r}-\omega_i t)}$, implies a wave frame of detection, regulated by interference effects in the detection.

It seems clear then that in the quantum theory of light detection, the particle and wave pictures are united as two sets of relative features of the same field in different frames of detection; thus they can be related to each other in such a way that Eq. (1) is left invariant – *the principle of relativity.* This unification can be characterized by a term called *particle-wave* rather than 'particle and wave', the hyphen emphasizing the new kind of unification.

It should be noted that in spite of the above-described unification of particle and wave pictures brought about in the quantum theory of detection, there remains a rather important and peculiar distinction between them, resulting from the fact that $\hat{a}_i$ and $\hat{a}_i^\dagger$ are operators but the phase factors $e^{i(\mathbf{k}_i\cdot\mathbf{r}-\omega_i t)}$ ($e^{-i(\mathbf{k}_i\cdot\mathbf{r}-\omega_i t)}$) are c-numbers. On the basis of this distinction, it also clear that the modulation by the phase factors in the probability expression of Eq. (4) at a velocity (the phase velocity) greater than c, for example, in de Broglie matter systems, in no way confuses us on the maximum speed of propagation of the signals, provided that a signal propagation is *physically* described by the annihilation and creation operators $\hat{a}_i$ and $\hat{a}_i^\dagger$.

One can conclude that the Newtonian analysis of the world into constituent objects has been replaced in terms of a kind of *interactive* pattern between the fields and their detection by the observer. The implications of this approach avoids much of our confusion in the wave-particle duality, if we regard the quantum theory of light detection as a kind of conceptual map of the *events* in the world, in a similar manner to the Minkowski diagram in Einstein's theory of special relativity. [1]

Because of the relativistic unification of the particle and wave pictures into the single expression of Eq. (1), there appears an illusion of co-existence of these two pictures. However, a little reflection shows that this view of the quantum theory of light detection is very far from the truth indeed. Consider, for example, that an observer wants to measure the speed of a light signal, then they must construct a particle frame of detection that registers where, and when, a light signal is emitted and then absorbed (we note that the propagation of a light signal is in fact what Einstein studied in the development of his special relativity theory). Such an observer cannot survey the whole of Eq. (1); they can only obtain the propagation details of the operators $\hat{a}_i$ and $\hat{a}_i^\dagger$. Therefore, the exact information of the phase factor $e^{i(\mathbf{k}_i\cdot\mathbf{r}-\omega_i t)}$ ($e^{-i(\mathbf{k}_i\cdot\mathbf{r}-\omega_i t)}$) is unknown to the observer; for this an interference experiment is required.

Thus, we suggest that the quantum theory of light detection can be envisioned as a conceptual map, having an invariant structure, containing the 'real' set of fields and their detection which can be observed experimentally. "In all maps (conceptual or otherwise) there arises the need for the user to locate and orient himself by seeing which point on the map represents *his* position and which line represents the direction in which *he is* looking" .[1] In doing this, one recognizes that every act of *actualization* yields a unique perspective on the world. But with the aid of the quantum theory of light detection, one can relate what is seen from one perspective (the particle frame) to what is seen from another (the wave frame). In this way one can abstract out what is invariant under a change of perspective, which leads to an ever-improving knowledge and understanding of the actual character of the radiation system under investigation. Therefore, when an observer, performing experiments with different frames of detection, is to understand what is observed, *he* need not puzzle, regarding which view is 'correct' and which view is 'wrong' (wave or particle). Rather, *he* consults the map provided by Eq. (1), and tries to come to a common understanding of why in each way detecting the same field has a different perspective. Different frames may be related to one another, for example, by employing the de Broglie relation, $p = h/\lambda$.

It is noted that the discussion of particle and wave pictures in terms of detection frames is rather similar to that in Bohm theory, [4,11–13] in which each picture, particle or wave, can be precisely defined, than in Heisenberg theory with indeterminacy. It should also be emphasized that the above discussion is confined to the electromagnetic field only. A generalization to other field theories is believed to reveal its further connection to Bohm theory.

In summary, according to Einstein, there is no 'absolute' notion necessary in physics, but rather, the task of physics is the study of *relationships* that are in principle observable. We have developed a similar relational approach to wave-particle duality, whereby the notion of *localizability*, which applies to the particle like simultaneity applies to time, is re-examined critically based on the detection facts. On the basis of the observed probabilistic law of detection events, one realizes that the concept of position is no longer valid in quantum theory. Consequently, by an analysis of the particle and wave notions in terms of *frames of detection* (c.f. space and time concepts in terms of frames of reference), the quantum theory of light detection is shown to express, in precise quantitative form, the relativity implied between the particle and wave pictures. The significance of this formalism in quantum physics is as a conceptual map of the events, similar to the Minkowski diagram in the theory of special relativity, of the world which contains the perspective of the observer implicitly. Thus, whether we consider what is seen by different observers or by the same observer in different frames, it is always necessary to relate the results of all these observations, by referring them to a particle-wave map with the correct structure. In this way one can understand what is invariant and therefore not dependent on the special perspective of each observer.

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